

Decoherence of collective atomic spin states due to inhomogeneous coupling

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We investigate the decoherence of a superposition of symmetric collective internal states of an atomic ensemble due to inhomogeneous coupling to external control fields. For asymptotically large system, we find the characteristic decoherence rate scales as \sqrt{N} with N being the total number of atoms. Our results shed new light on attempts for quantum information processing and storage with atomic ensembles.

Coherent quantum information encoding and processing has recently emerged as a major goal for the physics community. Despite the seemingly insurmountable difficulties, a rich variety of implementations are being pursued in laboratories across the globe. Among the early success is the apparent ability in simulating quantum operations with liquid based NMR [1, 2]. Recent theoretical efforts indicate, however, that in the pseudo-pure state approach using NMR, *quantum entanglement*, a key element for powerful quantum information processing, was in fact no present [3, 4]. Room temperature NMR technique, is therefore limited and can not be explored fully to benefit from an exponentially large Hilbert space of only a polynomially scaled resources and controls [5]. Nevertheless, early NMR based experiments have provided useful insight into the operations of genuine quantum computers [6].

Over the last few years, using symmetric collective internal states of an atomic ensemble, has attracted much attention [7, 8, 9, 10, 11]. The pros and cons of such an approach assisted with cavity photon-atom interaction was recently discussed by Fleischhauer *et al.* [9, 10]. In normal cavity QED based quantum computing implementations, atomic qubits are entangled and logic operations performed through their interaction with the common cavity photon quantum field. To maintain quantum coherence, it is important to reach the so-called strong coupling regime, when the single-photon coherent coupling $g_0 \gg \gamma, \kappa$, the atomic and cavity dissipation (decoherence) rates respectively [12]. The symmetric collective internal states can reach the strong coupling regime without requiring a high finesse cavity as $g_0 \propto \sqrt{N}$, with N the total number of atoms [13, 14, 15]. When implemented with protocols insensitive to individual atomic dissipation/decoherence rate as in the Dark state based adiabatic transfer protocols [7, 8, 16], one can apparently gain as upper hand over systems based on single atoms inside a cavity [9, 10]. This is in fact, not quite surprising, earlier cavity QED experiments have relied on the enhanced dipole interaction of a collection of many atoms [13, 14, 15]. In free space, the phenomena of superfluorescence or super-radiance [13, 17] constitutes another example of collective state dynamics. Recent experimental success clearly demonstrates the power of such an atomic ensemble based system for entangling macroscopic objects [18, 19]. Several new ideas have raised further ex-

pectation of exciting developments to come [20, 21]. Nevertheless, all ensemble based systems suffer from the reduced size of computational Hilbert space. In this case of symmetric collective internal states, the space used for quantum information, is much less than the 2^N for N two level atoms [22]. In view of the recent experimental success in storage and recovery of light coherence in atomic gases [23, 24], a related question to address is the sensitivity to errors when collective spin states are as quantum memories.

In this paper, we investigate the decoherence for a superposition of symmetric internal states of an atomic gas due to its inhomogeneous coupling with external control fields. Our study is motivated by the simple observation that the symmetric states of an atomic ensemble spans the computation space only if atoms can be manipulated cooperatively, namely, the coupling of both the external manipulating field and the environment surrounding the atomic ensemble should be homogeneous such that the collective motion of the atomic ensemble can be described by the collective quasi-spin operators. In essence the effect of different spatial positions for atoms $1, 2, \dots$, and N , is ignored or absorbed into each single spin operators. In reality, an optically thick atomic ensemble suffers from inhomogeneous coupling to both classical and quantum light fields, i.e. the coupling strength is position dependent. Such a situation arises naturally for trapped ions due to its center of mass motion. In this case, it is well known that the loss of quantum coherence for a superposition of internal state occurs. In this study, we refer such decoherence effect as inhomogeneous decoherence. We focus on introducing our technique and study the simplest example of superpositions of collective atomic Dicke states in this paper. The consideration of Dark-state, polariton approach based proposals [10] will be given in the future, as significant complications arise when the quantum cavity field is included.

Our model constitutes of an ensemble of two-level atoms described by Hamiltonian

$$H = \sum_{k=1}^N \left[\frac{1}{2} \omega_a^{(k)} \sigma_z^{(k)} + \frac{1}{2} (g_0^{(k)} \sigma_+^{(k)} + h.c.) \right], \quad (1)$$

where the σ 's are the standard Pauli matrices ($\hbar = 1$), and the different local coupling $g_0^{(k)}$ (for the k -th atom) may be due to a cavity mode profile as common in tightly

focused cavities or when atomic motional wave packet is insufficiently localized [25]. $\omega_a^{(k)} = \varepsilon_a^{(k)} - \omega_L$, is the difference between atomic energy $\varepsilon_a^{(j)}$ and the external near resonant laser frequency ω_L . For convenience, we further abstract Eq. (1) into the compact form $H = \sum_{k=1}^N \vec{B}^{(k)} \cdot \vec{\sigma}^{(k)}$, with real parameters $B_\mu^{(k)}$.

The symmetric collective spin space \mathcal{V}^S of dimension $2J+1 \ll 2^N$ ($J = N/2$) is spanned by the collective angular momentum states $\{|J, M\rangle, M = -J, \dots, J-1, J\}$ of $J_\mu = \sum_{i=1}^N \sigma_\mu^{(i)}/2$ satisfying $[J_\mu, J_\nu] = i\epsilon_{\mu\nu\zeta} J_\zeta$ and $J_x^2 + J_y^2 + J_z^2 = J^2 = J(J+1)$. $\epsilon_{\mu\nu\zeta}$ is the symmetric permutation tensor. The $|J, M\rangle$ space can be generated from the ladder operator $J_\pm = J_x \pm iJ_y$ according to [26],

$$|J, M\rangle = \sqrt{\frac{(J-M)!}{(J+M)!(2J)!}} J_+^{J+M} |J, -J\rangle, \quad (2)$$

except we note that an arbitrary unimodular phase can be self-consistently included with $J_\pm = \sum_{k=1}^N e^{\pm i\theta_k} \sigma_\pm^{(k)}/2$ and $|J, -J\rangle = |\downarrow, \downarrow, \dots, \downarrow\rangle$.

For any realistic system, an inhomogeneous distribution of the parameter $\vec{B}^{(j)}$ makes it impossible to constrain the system dynamics within the subspace \mathcal{V}^S . To facilitate further discussion denote $H = H_0 + H_1$ with $H_0 = \sum_{k=1}^N \vec{B} \cdot \vec{\sigma}^{(k)}$ and $H_1 = \sum_{k=1}^N \vec{b}^{(k)} \cdot \vec{\sigma}^{(k)}$, where $\vec{B}^{(k)} = \vec{B} + \vec{b}^{(k)}$ with $\vec{B} = \sum_k \vec{B}^{(k)}/N$. H_0 constitutes the intended coupling between the symmetric collective spin states, while H_1 represents a source of inhomogeneous decoherence. It causes decoherence as it provides a direct coupling from the subspace \mathcal{V}^S to its complement \mathcal{V}^O in \mathcal{V}^T . A quantitative measure for the unwanted coupling H_1 is in terms of the leakage parameter. Suppose initially the system is prepared in a superposition of collective spin states $|\phi(0)\rangle \in \mathcal{V}^S$. The intended dynamics governed by $U_0(t) = e^{-itH_0} = \prod_{k=1}^N e^{-it\vec{B} \cdot \sigma^{(k)}}$ leads to the resultant state $|\phi(t)\rangle_0 = U_0(t)|\phi(0)\rangle$, still within the same subspace. The actual final state is $|\phi(t)\rangle = U(t)|\phi(0)\rangle$ with $U(t) = \prod_{k=1}^N e^{-it\vec{B}^{(k)} \cdot \sigma^{(k)}}$ generally will span more than \mathcal{V}^S . The leakage can therefore be defined as

$$\xi = 1 - |\langle\phi_0(t)|\phi(t)\rangle|^2. \quad (3)$$

$\xi = 0$ corresponds to no leakage, while $\xi \rightarrow 1$ indicates a complete loss of the system coherence and population.

Denote $|\phi(0)\rangle \in \mathcal{V}^S$ as a normalized state expanded in terms of $|J, M\rangle$,

$$|\phi(0)\rangle = \sum_{M \ll N, \text{ or } M \sim N} c_M |J, M\rangle, \quad (4)$$

the overlap

$$\begin{aligned} |\langle\phi_0(t)|\phi(t)\rangle|^2 &= |\langle\phi(0)|U_0^\dagger(t)U(t)|\phi(0)\rangle|^2 \\ &= \sum_M \sum_{M'} c_{M'}^* c_M O_{M'M}(t) \leq 1, \end{aligned} \quad (5)$$

becomes the focus of our study with

$$\begin{aligned} O_{M'M}(t) &\equiv \langle J, M' | U_0^\dagger(t) U(t) | J, M \rangle \\ &= \langle J, M' | \prod_{k=1}^N O^{(k)} | J, M \rangle, \end{aligned} \quad (6)$$

$O^{(k)} = R^{(k)} + i\vec{I}^{(k)} \cdot \vec{\sigma}^{(k)}$, and

$$\begin{aligned} R^{(k)} &= \cos Bt \cos B^{(k)}t + (\hat{n} \cdot \hat{n}^{(k)}) \sin Bt \sin B^{(k)}t, \\ \vec{I}^{(k)} &= \hat{n} \sin Bt \cos B^{(k)}t + \hat{n}^{(k)} \cos Bt \sin B^{(k)}t \\ &\quad + (\hat{n} \times \hat{n}^{(k)}) \sin B \sin B^{(k)}t. \end{aligned} \quad (7)$$

We have defined $\hat{n} = \vec{B}/B$ and $\hat{n}^{(i)} = \vec{B}^{(i)}/B^{(i)}$.

The evaluation of Eq. (6) is difficult as state $|J, M\rangle$ involves a symmetric permutation of all atoms so that the \prod_k factor can not be pulled outside the inner product. Furthermore, $\prod_{k=1}^N O^{(k)}$ expands into 2^N separate terms, involving asymmetric products of $\vec{\sigma}^{(k)}$ of upto powers of N . A similar product structure was found to be responsible for decoherence in quantum measurement models [27], where the decoherence factor (the overlaps of the final states of detector or a environment) suppresses the off-diagonal element of its reduced density matrix. In mathematical terms, for a factorized state $|f\rangle = \prod_{k=1}^N |f^{(k)}\rangle$, the overlap integral $\langle f | \prod_{k=1}^N W_{M'M}^{(k)} | f \rangle$ becomes $\prod_{k=1}^N \langle f^{(k)} | W_{M'M}^{(k)} | f^{(k)} \rangle$, which approaches zero in the limit of macroscopic N as each factor $\langle f^{(k)} | W_{M'M}^{(k)} | f^{(k)} \rangle$ has a norm less than unity. To make a similar argument for the present problem, we need to find an expression such that the collective state $|J, M\rangle$ becomes factorized. Since we are interested in obtaining the asymptotically valid results in the limit of large N , a short time approximation (small t) can not be simply adopted. Following early discussions on atomic coherent states [28, 29], we introduce

$$|\theta\rangle = \prod_{k=1}^N \frac{1}{\sqrt{2}} \left(1 + \frac{e^{i\theta}}{2} \sigma_+^{(k)} \right) |\downarrow\rangle = \frac{1}{2^{N/2}} e^{J+e^{i\theta}} |J, -J\rangle, \quad (8)$$

a phase coherent state, that can be expanded according to the number of excitations

$$\begin{aligned} |\theta\rangle &= \frac{1}{2^{N/2}} \left[1 + e^{i\theta} J_+ + \dots + \frac{e^{in\theta}}{n!} J_+^n + \dots \right] |J, -J\rangle \\ &= \sum_{M=-J}^J \frac{e^{i(J+M)\theta}}{\mathcal{N}_{JM}} |J, M\rangle, \end{aligned} \quad (9)$$

where $\mathcal{N}_{JM} = \sqrt{(J+M)!(J-M)!2^N/(2J)!}$. The inverse transformation gives

$$|J, M\rangle = \frac{\mathcal{N}_{JM}}{2\pi} \int_0^{2\pi} e^{-i(J+M)\theta} |\theta\rangle d\theta, \quad (10)$$

which helps to evaluate Eq. (6) as $O_{M'M} = \mathcal{N}_{JM} \mathcal{N}_{JM'} o_{M'M}$ with the reduced overlap

$$o_{M'M} = \frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_0^{2\pi} d\theta'$$

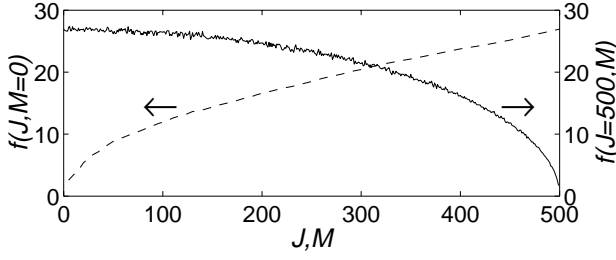


FIG. 1: M and J dependence of f for $J = 500$ and $B_z = 1$.

$$e^{-i(J+M)\theta} e^{i(J+M')\theta'} \prod_{k=1}^N G^{(k)}(\theta, \theta') \quad (11)$$

in a simple factorized form and

$$G^{(k)} = \frac{1}{2} {}_k\langle \downarrow | (1 + e^{-i\theta'} \frac{\sigma_{+}^{(k)}}{2}) O^{(k)} (1 + e^{i\theta} \frac{\sigma_{+}^{(k)}}{2}) | \downarrow \rangle_k.$$

$|G^{(k)}| \leq 1$ as both $(1 + e^{i\theta} \frac{\sigma_{+}^{(k)}}{2}) | \downarrow \rangle_k / \sqrt{2}$ and $(1 + e^{-i\theta'} \frac{\sigma_{+}^{(k)}}{2}) | \downarrow \rangle_k / \sqrt{2}$ are normalized. This points to a strong physical argument against rapid decoherence of collective spin state qubits. The question to answer is now clearly how does O_{MM} approach 0 due to inhomogeneous coupling. If the coupling coefficients $g_0^{(k)}$ and $\omega_a^{(k)}$ were constants (independent of atom label k), $O_{MM'} \equiv \delta_{MM'}$.

We investigate the above question for several model cases. First, we look at inhomogeneous broadening when $B_x^{(k)} = B_y^{(k)} = 0$ and $B_z^{(k)}$ satisfies a normal distribution (with respect to k) with mean $\bar{B} = B_z \hat{z} = \langle B_z^{(k)} \rangle \hat{z}$, and variance σ_z^2 . We find $O_{MM'}(t) \propto \delta_{MM'}$ with the coefficient being a constant unity for $|M| = J$ but decays with a time constant $T_{1/2} \propto 1/(\sqrt{N}\sigma_z)$ for $|M| < J$. Define $T_{1/2} \equiv 1/(f\sigma_z)$, we find f is essentially independent of σ_z for $\sigma_z \in [10^{-7}, 10^{-1}]B_z$. It contains an apparent dependence on $J^2 - M^2$ as shown in Fig. 1 for a given J and B_z . The J dependence (for $M = 0$) is also shown in the same figure. Based on our extensive numerical study, we find to a high level of accuracy

$$T_{1/2}(J, M, \sigma_z) = \frac{1}{\kappa \sigma_z \sqrt{J} \sqrt{1 - M^2/J^2}}, \quad (12)$$

with κ (≈ 1.2), essentially independent of B_z for $B_z \in [10^{-2}, 10^2]$.

This result is to be expected based on the collapse and revival of a quantum wave packet [30], since each individual atom collapses with a time constant $\propto 1/\sigma_z$, the collective states of an Gaussian ensemble should collapse with a time constant $\propto 1/(\sqrt{N}\sigma_z)$ as the net variance simply adds. This is indeed what we find for $M = 0$ or in general for $|M| \ll J$. Equation (12) also indicates that significantly reduced decoherence does occur in this case for $|M| \sim J$, a regime where collective spin states are mostly useful [7, 8, 9, 10, 20, 21]. In fact, for a single

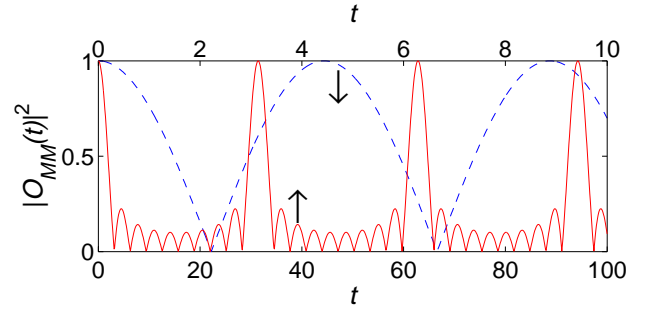


FIG. 2: Periodic behavior for $|O_{MM}(t)|^2$. Solid line denotes $M = 4$ and $N = 10$ with respect to the lower time axis, while dashed line denotes $M = 2$ and $N = 2$; $B_z^{(k)} = k$ is taken for $N = 10$ to assure the appearance of revival. For $N = 2$ revival occurs for arbitrary random values of $B_z^{(k)}$.

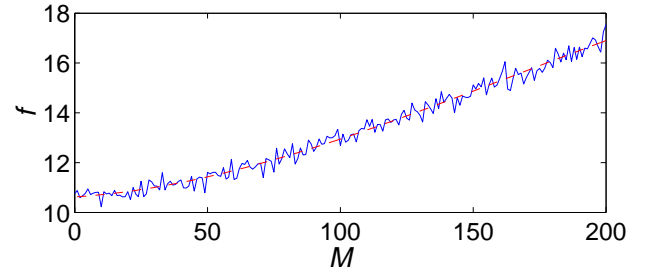


FIG. 3: The M dependence of f for $J = 200$ and $B_r = 10$. The smooth curve is a fit given by $-4.06483 \times 10^{-7}|M|^3 + 2.03393 \times 10^{-4}M^2 + 0.00697|M| + 10.62188$.

qubit quantum memory involving the two state superposition of $M = -J$ and $-J + 1$, the decoherence rate is just that of a single atom [9]. For small values of N , when ratios of different coupling strength $B_z^{(k)}$ match ratios of integers, we indeed were able to find the expected revival as shown in Fig. 2. This of course will not happen for an ensemble with a macroscopic N .

Next we consider the case of inhomogeneous Rabi coupling with $B_{x/y}^{(k)}$ being Gaussian distributions with mean $B_x = B_y = B_r$ and variance $\sigma_x^2 = \sigma_y^2 = \sigma_r^2$, and $B_z^{(k)} = 0$. Similar to the previous case, we find the diagonal term $O_{MM}(t)$ (including $M = \pm J$) decays with a time constant $T_{1/2} = 1/f\sigma_r$. The J dependence of $T_{1/2}$ is in fact almost identical, i.e. $f_{M=0} = \kappa_1 J^{1/2}$, with $\kappa_1 \approx 0.76$ when $B_r = 10$. The M dependence, on the other hand is more complicated as shown in Fig. 3. Obviously f does not depend on M linearly as now $|O_{MM}(t)|$ seems to decay faster for larger values of $|M|$.

The off-diagonal element $O_{M \neq M'}(t)$ grows to significant nonzero values, as shown by the typical sampling of $|O_{MM'}(t)|^2$ in Fig. 4 when N is not too large. Overall, we find the dependence on the random number sampling is strong only when $M - M' = \pm 1$, so we focus on $M - M' = \pm 2$ here. Define T_{\max} as the time for $|O_{MM'}(t)|$ to reach its first maximum and \mathcal{O}_{\max} the value

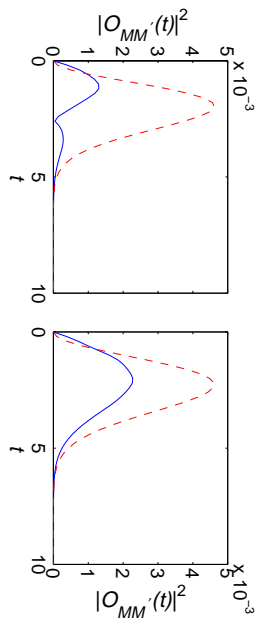


FIG. 4: Typical sampling of $O_{MM'}(t)$ for $J = 100$, $M = J - 1$, and $M' = M - 1$ (solid line) and $M - 2$ (dashed line). Different figures correspond to different random number sets.

of the maximum. We find that similar to the diagonals, $T_{\max} = 1/f\sigma_r$, with f a function of J, M, M' , and B_r , although O_{\max} seems to be largely independent of σ_r . To study the J dependence of f and O_{\max} , we consider the limiting case of $|M| \sim J$ when collective states are usual proposed to work. The result $f \propto J^{1/2}$ is once again as expected. In this case, we also find quite accurately $O_{\max} \propto J^{-1}$.

To summarize, we find within our model, the apparent decoherence or dissipation rate for superpositions of collective spin states scales as \sqrt{N} . This evidence clearly demonstrates that asymptotically there is no advantage of using collective spin states for quantum information processing. The \sqrt{N} enhanced coherent dynamics is simply being compensated by the \sqrt{N} enhanced decoherence when inhomogeneous coupling arises.

Finally, we note that our result also applies to the case of entangled states between the collective spins of two separate ensembles. For instance, for two ensembles A and B , a state $\sum_{M_A, M_B} c_{M_A, M_B} |J_A, M_A\rangle_A |J_B, M_B\rangle_B$ can always be expressed as coherent superposition of the total angular momentum basis $\vec{J} = \vec{J}_A + \vec{J}_B$, i.e. into collective basis $|J_A, J_B; J, M = M_A + M_B\rangle$ with $J = J_A + J_B = (N_A + N_B)/2$.

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